Problem of the week

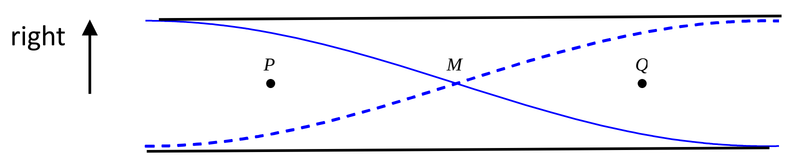
Standing waves and resonance

(a) Describe how a standing wave is formed.

(b) State two differences between a standing wave and a travelling wave.

(c) A loudspeaker is placed near the open end of a pipe whose other end is closed. Outline why a standing wave will be established in the pipe only for specific frequencies of sound.

(d) The diagram shows a standing wave established in a pipe with both ends open. The solid line represents the standing wave at t = 0 and the dashed line at $t = \frac{T}{2}$ where T is the period. P and Q are the equilibrium positions of two particles in the pipe.



State and explain whether the middle of the pipe, M, is the center of a compression or a rarefaction or neither at

(i) $t = 0$,

(ii) $t = \frac{T}{2}$.

(iii) Draw arrows to show the velocities of P and Q at $t = \frac{T}{4}$.

(iv) State the phase difference between P and Q.

(e) Two consecutive harmonics in a pipe with one open and one closed end have frequencies 270 Hz and 378 Hz.

(i) Determine the frequency of the first harmonic in this pipe.

(ii) The length of the pipe is 1.5 m. Estimate the speed of sound.

(f) In driven oscillations a system with damping has constant amplitude. Suggest what this implies for the rate at which energy is supplied to the system by the driving force.

(g) The natural frequency of an oscillating system is 12 Hz. The equation giving the displacement of the system from equilibrium is $x = x_0 \sin(28\pi t + \phi)$.

(i) Suggest why the system is been driven by an external periodic force.

(ii) The driving frequency is changed to 16 Hz. State and explain the effect of this, if any, on the amplitude of oscillations.

Answers

(a) A standing wave is formed when two identical travelling waves moving in opposite directions meet and superpose.

(b) Two from:

- A standing wave does not transfer energy, a travelling wave does.
- Different points in a standing wave have different amplitudes, in a travelling wave the amplitude for each point is the same.
- The phase of points within a loop is the same, in a travelling wave it varies.
- In a standing wave there are points nodes0 where the displacement is always zero , no such points exist in a travelling wave.

(c) The closed end must be a node and the open end an antinode. Thus, the possible wavelengths are $\lambda = \frac{4L}{n}$ where L is the length of the pipe and n is an odd integer. The frequency is thus $f = \frac{c}{4L}n$ and forms a discrete set of allowed frequencies.

(d)

(i) At $t = 0$, points to the left of M have displacements to the right i.e. closer to M and points to the right negative displacements i.e. closer to M. hence this is the center of a compression.

(ii) The reverse is true at $t = \frac{T}{2}$ making M the center of a rarefaction.

(iii) P to the left and Q to the right.

(iv) π

(e)

(i) $270 = \frac{c}{4L}n$ and $378 = \frac{c}{4L}(n+2)$. Hence $378 - 270 = 108 = \frac{c}{4L}(n+2) - \frac{c}{4L}n = \frac{c}{2L}$. The first harmonic has frequency $\frac{c}{4L} = \frac{108}{2} = 54 \text{ Hz}$

OR

270 and 378 are consecutive odd multiples of 54 (5 and 7)

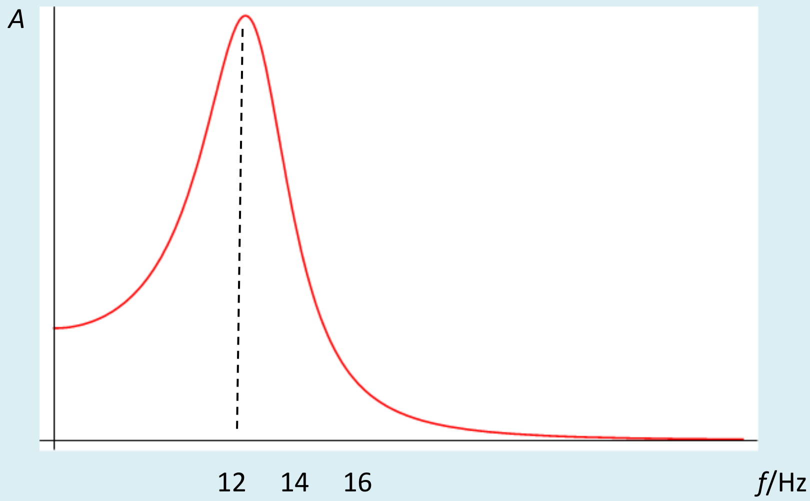
(ii) $f_1 = \frac{c}{4L} \Rightarrow 54 = \frac{c}{4 \times 1.5} \Rightarrow c = 324 \approx 320 \text{ m s}^{-1}$

(f) The rate at which energy is supplied to the system by the driving force is equal to the rate at which energy is being dissipated by the frictional forces.

(g)

(i) The frequency at which the system oscillates is 14 Hz and not the natural frequency of the system so an external force of frequency 14 Hz must act on the system.

(ii) From



the amplitude will decrease.